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Manifestations of Quantum Gravity in Scalar QED Phenomena

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Abstract.

Quantum gravitational corrections to the effective potential, at one-loop level and in the leading-log approximation, for scalar quantum electrodynamics with higher-derivative gravity —which is taken as an effective theory for quantum gravity (QG)— are calculated. We point out the appearence of relevant phenomena caused by quantum gravity, like dimensional transmutation, QG-driven instabilities of the potential, QG corrrections to scalar-to-vector mass ratios, and curvature-induced phase transitions, whose existence is shown by means of analytical and numerical study.

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As is known, Einstein gravity is non-renormalizable [1] and, therefore, it cannot play the role of a fundamental theory of quantum gravity (QG). In the best case, it might be regarded as an effective model of some consistent theory of QG, still unknown to us (for a recent discussion see [2]). In principle, one is allowed to consider alternative candidates as effective QG models (for an introduction to effective theories see [3]), as the effective theory for the conformal factor, whose aim is to describe QG in the far infrared limit [4], among others.

A natural idea is to add higher-derivative terms to the Einstein gravity action. In this way we obtain a higher-derivative QG which may be envisaged as an effective theory for QG on the same level as the Einsteinian one. This theory is known to be multiplicatively renormalizable [5], even in the presence of matter [6] (for a review, see [7]), and also asymptotically free. (Due to its perturbative non-unitarity this theory cannot be considered as an eligible candidate for a fundamental QG model but it is perfectly valid as an effective theory).

Since higher-derivative QG contains not only dimensional couplings —Newton's $16\pi G$ and the cosmological Λ— but also dimensionless coupling constants, one could expect some qualitative differences in its influence on physics near the Planck-mass scale ($\mu_{\rm Pl} \simeq 1.2 \cdot 10^{19}$ GeV) or between the Planck-mass and the GUTs energy scales ($\mu_{\rm GUT} \simeq 10^{15}$ GeV), as compared to the Einstein theory. In particular, there appear QG radiative corrections to the beta functions of the matter dimensionless coupling constants [6, 7]. It is precisely the purpose of the present letter to study QG corrections to one-loop and renormalization group (RG) improved effective potentials (EP) of massless scalar QED interacting with R^2-gravity. In this example we will show the existence of a variety of interesting phenomena: dimensional transmutation due to QG, corrections to scalar-to-vector mass ratios, QG-driven instabilities of EP, and curvature-induced phase transitions.

Let us start from the multiplicatively renormalizable theory with the Lagrangian [7]

$$\begin{aligned} \mathcal{L} &= \frac{1}{\lambda}W - \frac{\omega}{3\lambda}R^2 + \frac{1}{2}\xi R\varphi^2 \\ &+\frac{1}{2}\left(\partial_\mu\varphi_1 - eA_\mu\varphi_2\right)^2 + \frac{1}{2}\left(\partial_\mu\varphi_2 - eA_\mu\varphi_1\right)^2 - \frac{1}{4!}f\varphi^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \end{aligned} \tag{1}$$

where $W = C_{\mu\nu\alpha\beta}C^{\mu\nu\alpha\beta}$ and $\varphi^2 = \varphi_i\varphi_i$. The λ, ω, ξ are gravitational couplings, and the gravitational, scalar and vector fields are quantized. One could also have started from the massive model (including then also the Einstein theory) but, since we are concerned with the

approximation where the background scalar satisfies $\phi^2 \gg m^2$, being m^2 the largest effective mass of the theory, massive terms are not essential for our purposes and it is enough to work with (1). Setting $\omega = 0, \xi = 1/6$ in (1), we obtain conformally-invariant gravity with scalar QED. Such a theory is multiplicatively renormalizable only when using the so-called special conformal regularization (for a discussion and a list of references, see [7]).

Next, we find the effective potential [8, 9] for (1) on a scalar-gravitational background. The result for the RG-improved potential (for details on this formalism see [8, 10], in flat space, and [11], in curved space) can be obtained as follows: using the linear curvature approximation and $\varphi^2 \gg |R|$, i.e. expanding V in powers of the curvature, up to linear terms, we arrive at

$$V = \frac{1}{4!} f(t)\varphi^4(t) - \frac{1}{2}\xi(t)R\varphi^2(t), \tag{2}$$

where the system for the effective coupling constants may be found as

$$\lambda(t) = \frac{\lambda}{1 + \frac{\alpha^2 \lambda t}{(4\pi)^2}}, \qquad \alpha^2 = \frac{203}{15}, \qquad e^2(t) = \frac{e^2}{1 - \frac{2e^2\lambda t}{3(4\pi)^2}},$$

$$\begin{aligned}
\frac{d\omega}{dt} = \beta_\omega \quad &= \quad -\frac{1}{(4\pi)^2}\lambda\left[\frac{10}{3}\omega^2 + \left(5+\alpha^2\right)\omega + \frac{5}{12} + 3\left(\xi - \frac{1}{6}\right)^2\right], \\
\frac{d\xi}{dt} = \beta_\xi \quad &\equiv \quad \beta_\xi^{(0)} + \Delta\beta_\xi, \\
& \quad\quad \beta_\xi^{(0)} = \frac{1}{(4\pi)^2}\left(\xi - \frac{1}{6}\right)\left(\frac{4}{3}f - 6e^2\right), \\
& \quad\quad \Delta\beta_\xi = \frac{1}{(4\pi)^2}\lambda\xi\left[-\frac{3}{2}\xi^2 + 4\xi + 3 + \frac{10}{3}\omega + \frac{1}{\omega}\left(-\frac{9}{4}\xi^2 + 5\xi + 1\right)\right], \\
\frac{df}{dt} = \beta_f \quad &\equiv \quad \beta_f^{(0)} + \Delta\beta_f, \\
& \quad\quad \beta_f^{(0)} = \frac{1}{(4\pi)^2}\left(\frac{10}{3}f^2 - 12e^2 f + 36e^4\right), \\
& \quad\quad \Delta\beta_f = \frac{1}{(4\pi)^2}\left[\lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right. \\
& \quad\quad\quad\quad \left. -\lambda f\left(5 + 3\xi^2 + \frac{33}{2\omega}\xi^2 - \frac{6}{\omega}\xi + \frac{1}{2\omega}\right)\right], \\
-\frac{1}{\varphi}\frac{d\varphi}{dt} = \gamma_\varphi \quad &\equiv \quad \gamma_\varphi^{(0)} + \Delta\gamma_\varphi, \\
& \quad\quad \gamma_\varphi^{(0)} = -\frac{3e^2}{(4\pi)^2}, \\
& \quad\quad \Delta\gamma_\varphi = \frac{1}{(4\pi)^2}\frac{\lambda}{4}\left(\frac{13}{3} - 8\xi - 3\xi^2 - \frac{1}{6\omega} - \frac{2\xi}{\omega} + \frac{3\xi^2}{2\omega}\right),
\end{aligned} \tag{3}$$

and $t = \frac{1}{2} \log \frac{\varphi^2}{\mu^2}$, with the mass parameter μ^2 taken to be of the order of $\mu_{\rm Pl}^2$. The QG effective coupling constants and QG corrections to the matter beta functions have been calculated in [6, 7]. The Landau gauge is used for the vector field and the harmonic gauge for the gravitational field. As one can see from (2), (3), the RG-improved EP is given in a rather non-explicit form, but can be unveiled after some numerical work.

In the conformally invariant version of (1), the eqs. for ω and ξ disappear, and the only changing pieces of eqs. (3) are [7, 6]

$$\alpha^2 = \frac{27}{2},$$

$$\Delta\beta_f = \frac{1}{(4\pi)^2}\left(\frac{5}{12}\lambda^2 - \frac{41}{8}\lambda f\right), \qquad \Delta\gamma_\varphi = \frac{1}{(4\pi)^2}\frac{27}{32}\lambda. \tag{4}$$

Before going on to the numerical study of (2), let us consider the much simpler one-loop EP, which can easily be obtained from (2). If we use Coleman-Weinberg normalization conditions [8], it reads

$$\begin{aligned} V^{(1)} \;\; = \;\; & \frac{1}{4!} f \varphi^4 + \frac{1}{48}(\beta_f - 4f\gamma_\varphi)\varphi^4 \left(\log\frac{\varphi^2}{\mu^2} - \frac{25}{6}\right) \\ & -\frac{1}{2}\xi R \varphi^2 - \frac{1}{4}(\beta_\xi - 2\xi\gamma_\varphi) R\varphi^2 \left(\log\frac{\varphi^2}{\mu^2} - 3\right), \end{aligned} \tag{5}$$

where all the beta functions have already been defined in (3). Expression (5), which is valid for any arbitrary massless renormalizable theory, was first obtained in [12]. For its conformal version, one just has to set $\xi = 1/6$, $\beta_\xi = 0$ and take the $\Delta\beta_f$ and $\Delta\gamma_\varphi$ given in (4).

First, let us work with $V^{(1)}$ on a flat background. Then, choosing $\mu^2 = \varphi_m^2$, where φ_m^2 is the minimum of the potential, from the condition $\frac{\partial V^{(1)}}{\partial \varphi} = 0$ one can define the scalar coupling in terms of the remaining ones. Supposing that the QG coupling λ^2 is larger than or of the same order as e^4 (otherwise, if the e^4-term is leading in β_f we are in the situation of Ref. [8]), we get (for $f \sim e^4 + \lambda^2$)

$$V_{\rm flat}^{(1)} = \frac{1}{48(4\pi)^2}\left[36e^4 + \lambda^2\xi^2\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right]\varphi^4\left(\log\frac{\varphi^2}{\varphi_m^2} - \frac{1}{2}\right). \tag{6}$$

This generalizes the dimensional transmutation mechanism [8] so as to include QG modifications. It also shows that *even at* $e^2 = 0$ there may exist dimensional transmutation induced by QG. From (6) one can easily find QG corrections for the scalar-to-vector mass ratio. After

shifting the field, the scalar meson mass is given by

$$m^2(S) = V_{\text{flat}}^{(1)\prime\prime}(\varphi_m). \tag{7}$$

The photon mass $m^2(V) = e^2\varphi_m^2$ appearing after spontaneous symmetry breaking does not change as compared with the case of no QG. Thus

$$\frac{m^2(S)}{m^2(V)} = \frac{1}{6(4\pi)^2}\left[36e^2 + \frac{\lambda^2\xi^2}{e^2}\left(15 + \frac{3}{4\omega^2} - \frac{9\xi}{\omega^2} + \frac{27\xi^2}{\omega^2}\right)\right]. \tag{8}$$

In the strong gravity regime this is dominated completely by the second term, i.e. by QG. One may expect that the analog of (8), where the QG part is universal, may influence the GUT phenomenology.

We can as well define the relation (8) for the conformal version of (1):

$$\frac{m^2(S)}{m^2(V)} = \frac{1}{6(4\pi)^2}\left(36e^2 + \frac{5}{12}\frac{\lambda^2}{e^2}\right). \tag{9}$$

QG corrections can again become dominant if $\lambda^2 \gg e^4$. Note that, taking into account the curvature terms in (5), it is also possible to define ξ in terms of e^2, λ ($\xi \sim e^2 + \lambda^2$) and find the curvature corrections to (8).

Now, we go on to discuss the stability of the EP. It is clear from expression (2) that the RG-improved potential is stable at large t if $f(t) \geq 0$. Without electromagnetic sector ($e^2 = 0$) and at extremely large t, $f(t)$ becomes negative (for the conformal version) and we observe the above mentioned QG-driven instability of the EP. The same phenomenon takes place in the general version, for reasonable choices of ξ and ω. However, when the electromagnetic sector is present, there is always stability for large t. (Note that in this case there exists also a limitation to the t-region, as t should be less than $t_p = \dfrac{3(4\pi)^2}{2e^2}$, corresponding to the Landau pole).

In Fig. 1 we show the behaviour of the scalar coupling constant for a case of the general theory (1). $f(t)$ becomes negative when approaching GUT scales and leads to instability of the RG-improved potential for $t < t_i \simeq -6$. In this t-region, the conformal version does not show any instability. Note that, since $\dfrac{1}{2}\log\dfrac{\mu_{\text{GUT}}^2}{\mu_{\text{Pl}}^2} \simeq -9$, the decrease of t from 0 to around –9 corresponds to changing the energy scale from μ_{Pl}^2 to μ_{GUT}^2.

We shall now examine another phenomenon resulting from QG, namely spontaneous symmetry breaking and phase transitions induced by curvature. It is known that in curved space-time spontaneous symmetry breaking may take place, in the massless case, already at the

classical level if $\xi R > 0$, $\varphi^2 = \frac{6\xi R}{f}$. Taking into account quantum corrections of matter fields in an external gravitational field, one can find the possibility of curvature-induced phase transitions (see [13], for QED in curved space, a general picture is given in [7, 12]). At this point, we will discuss the same phenomenon for the RG-improved EP in the presence of QG.

Fig. 2 displays typical shapes of the EP in the symmetry breaking phase for the conformal version. This phase corresponds to positive curvature, and the one with unbroken symmetry to $R = 0$ —or also to negative R, when taken into consideration. In this region (below $\mu_{\rm Pl}$), the behaviour of the one-loop potential (for simplicity we keep only the logarithmic terms in the one-loop corrections) practically coincides with the behaviour of the RG-improved EP.

It is interesting to estimate the values of the induced cosmological and Newton couplings in the symmetry breaking phase. In particular, we may start from R^2-gravity with matter of the form (1) without Einstein and cosmological terms, at energies slightly below the Planck mass. Then, for the choice of parameters in Fig. 2, we estimate the values of the effective induced cosmological and Newton constants, which turn out to be the following (at curvatures $R = 10^{-4}$, which is higher than the typical curvature values in the GUT epoch, i.e. corresponding to the region between $\mu_{\rm GUT}$ and $\mu_{\rm Pl}$):

$$\frac{1}{16\pi G_{\rm ind}} \simeq 8.2 \cdot 10^{-4} \mu^2, \qquad \frac{2\Lambda_{\rm ind}}{16\pi G_{\rm ind}} \simeq 4.2 \cdot 10^{-8} \mu^4, \tag{10}$$

where $\mu_{\rm GUT} < \mu < \mu_{\rm Pl}$. Similar relations are true in the conformal version. Such an induced cosmological constant is too large as compared with experiment, but this may be reasonably compensated by the ensuing decrease of its value due to vacuum energy contributions at smaller scales.

Curvature-induced phase transitions at some given critical curvature R_c can be found also, although they appear for seemingly small values of R_c, corresponding to the range between the SM and GUT scales. Fig. 3 depicts the behaviour of RG-improved (solid line) versus one-loop (dashed line) potentials for the general theory. The first-order phase transition for the RG-improved potential corresponds to $R_c \simeq -10^{-9}\mu^2$.

To summarize, we have investigated in our simple model of scalar QED with quantum R^2-gravity the QG corrections to the effective potential and their influence on physically measurable phenomena, such as the vacuum structure, mass ratios, etc. By virtue of the universality of the

QG corrections, our formalism may certainly be applied to more realistic (and complicated) GUTs. Many questions can be studied in the framework of our formalism for those theories, like the stability of the scalar sector, the inducement of Einstein gravity, clearer conections with GUT phenomenology, etc. In particular, an interesting possibility is linked with the idea of an inflationary universe (for a review, see [14]) where inflation based simply on the Coleman-Weinberg symmetry breaking mechanism in GUTs is considered to be unrealistic. It would be of interest to understand how QG corrections influence the inflationary scenario.

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Figure captions

Fig. 1. Running scalar coupling $f(t)$, in the general theory with $\xi = 1$, $\omega = 1$. for different initial values of $\lambda(t = 0) \equiv \lambda$. We take $f(t = 0) \equiv f$ to be $e^4 + \lambda^2$, where $e \equiv e(t = 0)$, for reasons explained in the text. When setting $\xi = \frac{1}{6}$, the situation is practically the same as in the conformal version, and the instability disappears.

Fig. 2. Potential curves for the conformal version. In this range, the one-loop and RG-improved potentials coincide to the extent that their associated curves completely overlap one another. Taking $\xi = 1/6$, $\omega = 1$ in the general theory, the curves are very close to the ones shown.

Fig. 3. One-loop (dashed line) and RG-improved (solid line) potentials for the general theory, with $\xi = 1$, $\omega = 1$, $e^2 = 10^{-2}$, $\lambda = 0.1$, $f = e^4 + \lambda^2$, for three different values of the curvature R. At the scales represented, there is already some quantitative disagreement between both approaches, particularly in the precise value of R_c, which would be of $\simeq -2.13 \cdot 10^{-9}$ at one loop and of $\simeq -1 \cdot 10^{-9}$ for the RG-improvement, but the nature of the overall picture still coincides in both cases.